Coherently manipulating cold ions in separated traps by their vibrational couplings

Miao Zhang[1] and L. F. Wei*[1,2,3]

[1]*Quantum Optoelectronics Laboratory, School of Physics and Technology, Southwest Jiaotong University, Chengdu 610031, China*

[2]*State Key Laboratory of Optoelectronic Materials and Technologies, School of Physics and Engineering, Sun Yat-sen University, Guangzhou 510275, China*

[3]*State Key Laboratory of Functional Materials for Informatics, Shanghai Institute of Microsystem and Information Technology, Chinese Academy of Sciences, Shanghai 200050 China*

(Dated: January 18, 2013)

Recent experiments [K. R. Brown, *et al*., Nature 471, 196 (2011); and M. Harlander, *et al*., Nature 471, 200 (2011)] have demonstrated the coherent manipulations on the external vibrations of two ions, confined individually in the separated traps. Using these recently developed techniques, we propose here an approach to realize the coherent operations, e.g., the universal quantum gates, between the separated ion-trap qubits encoded by two internal atomic states of the ions. Our proposal operates beyond the usual Lamb-Dicke limits, and could be applied to the scalable ion-traps coupled by their vibrations.

PACS numbers: 03.67.Lx, 42.50.Dv, 37.10.Ty.

In the past two decades, much attention has been paid to the study of quantum computer, which uses the principles of quantum mechanics to solve problems that could never be solved by any classical computer [1]. Up to now, several kinds of physical systems, e.g., trapped ions [2], cavity QEDs [3], nuclear magnetic resonance [4], Josephson junctions [5], and coupled quantum dots [6], etc., have been proposed to implement the desirable quantum computation. Specifically, the system of trapped ions has many advantages such as convenient manipulation, relatively long coherence time, and easy readout [7].

Quantum computation with trapped ions was proposed by Cirac and Zoller [2]. In their scheme, a string of ions is trapped in *a single potential well*, and the ions' collective motions (CMs) act as the data bus to couple the distant qubits encoded by the internal atomic states of ions. The traditional Cirac-Zoller gate [2] requires that the CMs of trapped ions should be cooled to their vibrational ground state; thus the implementation of such a gate is practically sensitive to the decoherence of the motional states. Subsequently, Milburn [8] and Mølmer and Sørensen [9] proposed alternative models of quantum computation with warm ions, which relax the stringent requirements on cooling vacuum state and addressing individual ion. Recently, there has been much interest in *multi-zone traps* for the implementation of a scalable quantum computing network [10, 11], in which ions are confined individually in separated traps and coupled by their Coulomb interactions. Recently this idea has been demonstrated experimentally [12, 13] by directly controlling trapping potentials (i.e., the voltages on the DC electrodes) to tune their external vibrations for resonances.

Following the above experimental demonstrations [12, 13], in this brief report we propose an approach to implement a typical coherent manipulation, i.e., a universal controlled-NOT (CNOT) gate, between the separated ion traps. It is well known that such a universal gate, assisted by arbitrary single-qubit rotations, can generate any quantum computing network [14]. In our approach, several laser pulses are applied to the trapped ions to exchange the information between the external and internal states of the ions. The switchable coupling between the separately trapped ions is achieved by adiabatically sweeping the trapping potentials. Here, the adiabaticity means that such a sweeping does not yield any quantum transition between the ions' vibrational levels. Our proposal operates beyond the usual Lamb-Dicke (LD) limit [15, 16] and could be scaled to more than two qubits.

First, we simply review how to realize the laser-induced coupling between the external and internal states of a single trapped ion. A single trapped ion (with mass M and charge q) has two degrees of freedom: the vibrational motion around the trap center and the internal atomic levels. We assume that the ion trap provides a pseudopotential such that the ion's oscillation frequency ν along the axial direction is much smaller than those along the radial directions [17, 18]. As a consequence, only the quantized vibrational motion along the axial direction is considered. For the ion's internal degrees of freedom we consider two atomic levels, e.g., the ground state $|g\rangle$ and the excited state $|e\rangle$, to encode a qubit. The Hamiltonian describing the two degrees of freedom of the ion reads

$$\hat{H}_0 = \hbar\nu(\hat{a}^\dagger\hat{a} + \frac{1}{2}) + \frac{\hbar}{2}\omega_a\hat{\sigma}_z, \quad (1)$$

where $\hat{a}^\dagger$ and $\hat{a}$ are the bosonic creation and annihilation operators of the external vibrations and $\hat{\sigma}_z = |e\rangle\langle e| - |g\rangle\langle g|$ is the Pauli operator of the qubit. The transition frequency ω_a is defined by $\omega_a = (E_e - E_g)/\hbar$, with E_g and E_e being the corresponding energies of the ground and excited states, respectively. It is well known that the coupling between the above two uncoupled degrees of freedom of the ion can be achieved by applying suitable classical lasers. This coupling can be descried by the effective laser-ion interaction [15, 16]

$$\hat{H}_i = \frac{\hbar\Omega}{2}(\hat{\sigma}_+ + \hat{\sigma}_-)e^{i\eta(\hat{a}+\hat{a}^\dagger) - i\omega_l t - i\vartheta_l} + \text{H.c.}, \quad (2)$$

*weilianfu@gmail.com

where Ω is the Rabi frequency describing the strength of the coupling between the applied lasers and the trapped ion, η is the LD parameter describing the strength of the coupling between the external and internal degrees of freedom of the ion, ω_l and ϑ_l are the effective frequency and initial phase of the applied laser beams, respectively, and $\hat{\sigma}_+ = |e\rangle\langle g|$ and $\hat{\sigma}_- = |g\rangle\langle e|$ are the usual raising and lowering operators, respectively.

If the LD parameter η is sufficiently small, then the usual LD approximation works well and thus, by neglecting the terms relating the higher order of η, $\exp\left[i\eta(\hat{a}+\hat{a}^\dagger)\right] \approx 1+i\eta(\hat{a}+\hat{a}^\dagger)$ [17, 18]. Beyond such a limit, the total Hamiltonian $\hat{H} = \hat{H}_0 + \hat{H}_i$ of the trapped ion can be generally written as

$$\hat{H}' = \frac{\hbar\Omega}{2} e^{-\eta^2/2}\hat{\sigma}_+ \sum_{n,m}^{\infty}\left[(-1)^{n+m} f_1 + f_2\right] \times \frac{(i\eta)^{n+m}(\hat{a}^\dagger)^n \hat{a}^m e^{i(n-m)\nu t}}{n!m!} + \text{H.c.} \tag{3}$$

in the interaction picture defined by the unity operator $\hat{U}_1 = \exp(-it\hat{H}_0/\hbar)$, where $f_1 = \exp[i(\omega_a+\omega_l)t + i\vartheta_l]$ and $f_2 = \exp[i(\omega_a-\omega_l)t - i\vartheta_l]$. Specifically, for the kth red-sideband excitations, i.e., $\omega_l = \omega_a - k\nu$ with $k = 0, 1, 2, \cdots$, the above Hamiltonian reduces effectively to

$$\hat{H}_r = \frac{\hbar\Omega}{2} e^{-\eta^2/2 - i\vartheta_l}(i\eta)^k \hat{\sigma}_+ \sum_{j=0}^{\infty} \frac{(i\eta)^{2j}(\hat{a}^\dagger)^j \hat{a}^{j+k}}{j!(j+k)!} + \text{H.c.} \tag{4}$$

in the rotating-wave approximation [i.e., neglecting the rapid oscillating terms in (3)]. The dynamics of the trapped ion ruled by Hamiltonian (4) is exactly solvable. Indeed, if the external vibration of the ion is initially in a Fock state $|m\rangle$ and the internal atomic state is in the ground sate $|g\rangle$ (or excited state $|e\rangle$), then the relevant state evolutions can be given by

$$\begin{cases} |m,g\rangle \longrightarrow |m,g\rangle, \ \ m < k, \\ |m,g\rangle \longrightarrow \cos(\Omega_{m-k,k}t)|m,g\rangle \\ \qquad\qquad +i^{k-1}e^{-i\vartheta_l}\sin(\Omega_{m-k,k}t)|m-k,e\rangle, \ \ m \geq k, \\ |m,e\rangle \longrightarrow \cos(\Omega_{m,k}t)|m,e\rangle \\ \qquad\qquad -(-i)^{k-1}e^{i\vartheta_l}\sin(\Omega_{m,k}t)|m+k,g\rangle, \end{cases} \tag{5}$$

with

$$\Omega_{m,k} = \frac{\Omega}{2}\eta^k e^{-\eta^2/2}\sqrt{\frac{(m+k)!}{m!}} \sum_{j=0}^{m} \frac{(i\eta)^{2j} m!}{(j+k)!j!(m-j)!}\,. \tag{6}$$

We now consider two ions are trapped individually in two potential wells separated by a distance d between them [12, 13], as shown in Fig.1. The Hamiltonian describing the external motions of the two ions reads

$$\hat{H}_{ex} = \hbar\nu_1(\hat{a}_1^\dagger\hat{a}_1 + \frac{1}{2}) + \hbar\nu_2(\hat{a}_2^\dagger\hat{a}_2 + \frac{1}{2}) + V, \tag{7}$$

with

$$V \approx -\frac{q_1 q_2}{4\pi\epsilon_0 d}\left[1 + \frac{z_1 - z_2}{d} - \frac{(z_1-z_2)^2}{d^2}\right] \tag{8}$$

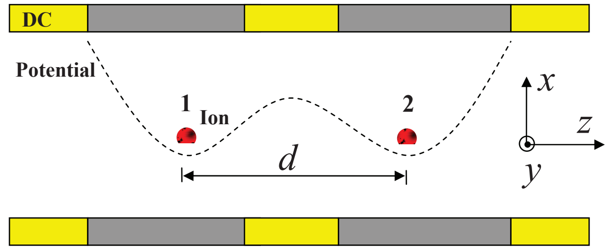

FIG. 1: (Color online) Sketch of two ions confined individually in two potential wells separated by a distance d between them.

being the Coulomb interaction between them. Here, z_1 and z_2 are the displacements of the trapped ions from the minimal values of the two potentials, respectively. In terms of the bosonic operators introduced above, the Coulomb interaction between the two ions can be rewritten as

$$V = \mathcal{K}\Bigg[\sum_{j=1}^{2} \frac{(-1)^j \xi_j}{d}\hat{a}_j + \frac{\xi_j^2}{d^2}(\hat{a}_j^2 + \hat{a}_j\hat{a}_j^\dagger) - \frac{2\xi_1\xi_2}{d^2}(\hat{a}_1\hat{a}_2 + \hat{a}_1\hat{a}_2^\dagger)\Bigg] + \text{H.c.}, \tag{9}$$

with $\mathcal{K} = q_1 q_2/(4\pi\epsilon_0 d)$ and $\xi_j = \sqrt{\hbar/(2M_j\nu_j)}$. Consequently, the Hamiltonian (7) can be further written as

$$\hat{H}'_{ex} = \mathcal{K}\Bigg[\sum_{j=1}^{2} \frac{(-1)^j \xi_j}{d}\hat{a}_j e^{-i\omega_j t} + \frac{\xi_j^2}{d^2}\hat{a}_j^2 e^{-i2\omega_j t} - \frac{2\xi_1\xi_2}{d^2}(\hat{a}_1\hat{a}_2 e^{-i(\omega_1+\omega_2)t} + \hat{a}_1\hat{a}_2^\dagger e^{i(\omega_2-\omega_1)t})\Bigg] + \text{H.c.} \tag{10}$$

in the interaction picture defined by unity operator $\hat{U}_2 = \exp(-it\hat{H}_{ex}^0/\hbar)$. Here, $\hat{H}_{ex}^0 = \sum_{j=1}^{2}\hbar\omega_j(\hat{a}_j^\dagger\hat{a}_j + 1/2)$, and $\omega_j = \nu_j + 2\mathcal{K}\xi_j^2/(\hbar d^2)$ is the renormalized frequency of the jth ion. Usually, the amending frequency $\tilde{\nu}_j = 2\mathcal{K}\xi_j^2/(\hbar d^2)$ is much smaller than the original frequency ν_j and thus could be negligible. For example, for the experimental parameters: $\nu_j = 2\pi \times 4.04$ MHz, $d = 40\,\mu$m, and mass $M_j = M(^9\text{Be}^+)$ [12], we have $\tilde{\nu}_j \approx 2\pi \times 1.5$ KHz.

Under the resonant condition, i.e., $\omega_1 = \omega_2$, the Hamiltonian (10) can be reduced effectively to

$$\hat{H}_{\text{eff}} = -\hbar g(\hat{a}_1\hat{a}_2^\dagger + \hat{a}_1^\dagger\hat{a}_2) \tag{11}$$

under the rotating wave approximation. Where, the coupling strength $g = 2\mathcal{K}\xi_1\xi_2/d^2$. Consequently, in the lowest two invariant subspaces $\{|0\rangle_1|0\rangle_2\}$ and $\{|0\rangle_1|1\rangle_2\rangle, |1\rangle_1|0\rangle_2\}$, the dynamics of the coupled bosonic modes reads

$$|0\rangle_1|0\rangle_2 \longrightarrow |0\rangle_1|0\rangle_2, \tag{12}$$

and

$$\begin{cases} |0\rangle_1|1\rangle_2 \longrightarrow \cos(gt)|0\rangle_1|1\rangle_2 + i\sin(gt)|1\rangle_1|0\rangle_2, \\ |1\rangle_1|0\rangle_2 \longrightarrow \cos(gt)|1\rangle_1|0\rangle_2 + i\sin(gt)|0\rangle_1|1\rangle_2. \end{cases} \tag{13}$$

Suppose that the two ions are initially trapped in the separated potential wells with different frequencies (i.e., without resonance), and their external vibrational states are both prepared in the vacuum state $|0\rangle$. Our approach, to realize the desirable CNOT gate between the internal atomic states of two ions, includes the following five sequential operations.

(I) A first red-sideband pulse with the duration $t_1 = \pi/(2\Omega_{0,1})$ and phase ϑ_1 is applied on the first ion to implement the operation $|0,e\rangle_1 \longrightarrow \exp[i(\vartheta_1+\pi)]|1,g\rangle_1$, leaving the state $|0,g\rangle_1$ unchanged.

(II) Varying adiabatically the trapping potentials such that the two vibrations are turned into resonance. After a duration $t_2 = \pi/(2g)$, this resonant coupling is then adiabatically turned off and thus the evolution $|1\rangle_1|0\rangle_2 \longrightarrow e^{i\pi/2}|0\rangle_1|1\rangle_2$ is generated, leaving the state $|0\rangle_1|0\rangle_2$ unchanged.

(III) A resonant pulse with the duration t_3 and phase ϑ_3 is applied on the second ion to implement the single-ion CNOT gate between the external and internal states of the ion [16]. This is the key step for generating the desirable CNOT gate between the two separately trapped ions. The implementation of such a single-ion CNOT gate (beyond the usual LD regime) by using a single resonant pulse will be further explained later in the text.

(IV) Repeating the operations in step (II).

(V) Another first red-sideband pulse with the duration $t_5 = \pi/(2\Omega_{0,1})$ and phase ϑ_5 is applied to the first ion.

After the above five sequential manipulations, we have

$$\begin{aligned}
&|0,g\rangle_1|0,g\rangle_2 \longrightarrow |0,g\rangle_1|0,g\rangle_2\\
&|0,g\rangle_1|0,e\rangle_2 \longrightarrow |0,g\rangle_1|0,e\rangle_2\\
&|0,e\rangle_1|0,g\rangle_2 \longrightarrow e^{i(\vartheta_1-\vartheta_3-\vartheta_5+3\pi/2)}|0,e\rangle_1|0,e\rangle_2\\
&|0,e\rangle_1|0,e\rangle_2 \longrightarrow e^{i(\vartheta_1+\vartheta_3-\vartheta_5+3\pi/2)}|0,e\rangle_1|0,g\rangle_2.
\end{aligned} \tag{14}$$

Certainly, if the phases of the applied laser beams satisfy the conditions $\vartheta_1-\vartheta_3-\vartheta_5 = \vartheta_1+\vartheta_3-\vartheta_5 = -3\pi/2$, then the qubits undergo a desirable CNOT logic gate and their vibrations return to the ground states.

We now discuss the conditions of the adiabatic manipulations in steps II and IV for switching on/off the ion-ion couplings. To realize these adiabatic operations, an possible method is to vary the vibrational frequency of one of the two ions, e.g., the first one, as $\nu_1(\tau) = \beta\tau + \nu_2$ with the rate β and time τ. This indicates that the detuning $\Delta = \nu_1(\tau) - \nu_2 = \beta\tau$ between the two vibrations of the ions are controllable (although the frequency ν_2 of the second ion is fixed). As a consequence, the coupling between the two ions could be turned on (with $\Delta = 0$) or off (with $\Delta \gg g$). Obviously, with the time-dependent frequency $\nu_1(\tau)$ the Hamiltonian (7) can be rewritten as

$$\hat{H}_{ex}(\tau) = -\frac{\hbar^2}{2m_1}\frac{\partial^2}{\partial z_1^2} + \frac{1}{2}M_1\nu_1^2(\tau)z_1^2 + \hbar\nu_2(\hat{a}_2^\dagger\hat{a}_2 + \frac{1}{2}) + V. \tag{15}$$

According to the standard quantum adiabatic theorem, the occupations of the vibrational states of the ions are unchanged during the adiabatic manipulations. For the present case, the condition

$$\gamma_{n,m} = \left|\frac{{}_1\langle n|\frac{\partial \hat{H}_{ex}(\tau)}{\partial\tau}|m\rangle_1}{\hbar\nu_1^2(\tau)(n-m)^2}\right| \ll 1, \quad n \neq m, \tag{16}$$

should be satisfied. Indeed, by proper setting the relevant parameters this condition can be well satisfied, e.g., $\gamma_{n,0} < 3.1\times10^{-6}$ and $\gamma_{n,1} < 5.3\times10^{-6}$ for the typical $\Delta = 100$ KHz, $\tau = 9\,\mu$s, and $\nu_1(\tau) = 2\pi\times4.04$ MHz [12]. Consider that two ^{9}Be$^+$ ions are trapped separately at the distance $d \approx 40\,\mu$m, and the resonant frequency is set at $\nu_1 = \nu_2 = 2\pi\times4.04$ MHz [12]. Then, the coupling strength is caculated as $g \approx 2\pi\times1.5$ KHz, which is far smaller than the initial detuning $\Delta = 100$ KHz set above. Obviously, the selected duration $\tau = 9\,\mu$s [12] of adiabatic operation is significantly shorter than the periods $2\pi/g \approx 0.67$ ms of the resonant coupling. This indicates that the influence of the adiabatic manipulation on the ion-ion coupling is practically weak. As a consequence, the influence on the designed π-pulse $\sin(\pi/2+2g\tau) \approx 0.99$, applied in steps II and IV, is practically weak, due to the relatively-fast adiabatic manipulations. Therefore, the present adiabatic manipulation is slow enough to satisfy the required adiabatic condition, but is practically sufficiently fast to realize the ion-ion switchable couplings.

The single-ion CNOT gate in step III is usually implemented within the LD limit, see, e.g., Ref [18]. In that experiment, three sequential laser pulses and an auxiliary atomic level are required. Beyond the LD limit and without the auxiliary level, we show in the step III that only a single laser pulse is sufficient to implement the desirable single-ion CNOT gate. Indeed, under the resonant driving, if the duration t_3 of the single laser pulse is set properly, e.g., $\cos(\Omega_{0,0}t_3) = \sin(\Omega_{1,0}t_3) = 1$, then the desirable CNOT gate of a single trapped ion could be implemented. Of course, the durations t_3 are now not that of the usual π-pulses [18] but other ones, which is resolved from $\cos(\Omega_{0,0}t_3) = \sin(\Omega_{1,0}t_3) = 1$ by the numerical method [16]. Using the Raman beams of experiment [17], the LD parameter $\eta \approx 0.33$ for the vibrational frequency $\nu = 2\pi\times4.04$ MHz, and $t_3 \approx 29.6\,\mu$s is resolved for the Rabi frequency $\Omega = 2\pi\times500$ KHz.

For the feasibility, the total durations t_{total} of the above five-step operations must be sufficiently short, such that the coherence of quantum states could not be disturbed significantly by the practically-existing decoherence. For the typical parameters listed above: $\eta = 0.33$, $\Omega = 2\pi\times500$ KHz, and $g = 2\pi\times1.5$ KHz, we have $t_1 = t_5 = \pi/(2\Omega_{0,1}) \approx 3.2\,\mu$s and $t_2 = \pi/(2g) \approx 166.7\,\mu$s. Consequently, the total duration $t_{\text{total}} = 2(t_1+t_2)+t_3+4\tau \approx 405.4\,\mu$s. This implies that the present approach to generate the desirable CNOT gate should be feasible, since the coherence of the vibrational states between the two trapped ions could be maintained on the order of milliseconds [12]. In principle, the above durations of operations can be significantly shortened by increasing the Rabi frequency Ω and the coupling strength g. Experimentally, these parameters can be enlarged by increasing the power P of the applied laser beams and decreasing the distance d between the potential wells (since $\Omega \propto \sqrt{P}$ and $g \propto 1/d^3$). For example, if P is enhanced to 5 mW (ten times than that used in experiment [17]) and d is shortened to $20\,\mu$m (half of that used in [12]), then the Rabi frequency $\Omega \approx 2\pi\times1.6$ MHz and the coupling strength $g \approx 2\pi\times12$ KHz can be reached. Thus, the total duration is shortened to $t_{\text{total}} \approx 88.9\,\mu$s.

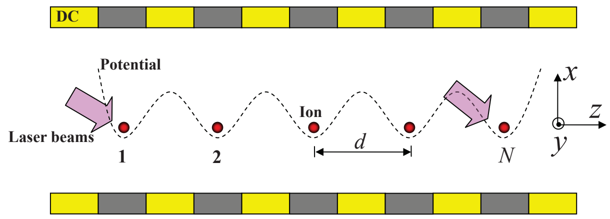

FIG. 2: (Color online) Sketch of coherent manipulations between two ions separated by the distance $(N-1)d$.

Finally, we discuss the scalability of the system, i.e., how many ions ($N > 2$) confined individually in different traps can be integrated as a trap-chain (see Fig. 2). For simplicity, we assume that the distances between the nearest-neighbor ions are uniform d, and all the ions are initially decoupled and cooled in their motional ground states $\prod_{j=1}^{N}|0_j\rangle$. As the distant coupling $g \propto 1/[(N-1)d]^3$ (between the first and Nth ions) decreases quickly with the increase of the integer number N, the quantum logic operation between them is not easy to be implemented by using such a direct coupling. A possibly feasible method is to sequentially implement the operation of resonant coupling between the nearest-neighbor ions one by one, i.e.,

$$|1\rangle_1|0\rangle_2\cdots|0\rangle_N|Q_N\rangle \longrightarrow e^{i\pi}|0\rangle_1|1\rangle_2\cdots|0\rangle_N|Q_N\rangle \longrightarrow \cdots \longrightarrow e^{i(N-1)\pi}|0\rangle_1|0\rangle_2\cdots|1\rangle_N|Q_N\rangle. \tag{17}$$

By this way the vibrational information of the first ion could be transferred to the last one without affecting their internal states (i.e., the N-qubit state $|Q_N\rangle$ is unchanged). This means that the step II should include $2(N-1)$ adiabatic operations for sequentially coupling the trapped ions one by one. Consequently, the total duration for generating the CNOT gate between the first and Nth ions is $t'_{\rm total} = 2\,[t_1+(N-1)t_2] + t_3 + 4(N-1)\tau \approx 709.7\,\mu$s with $\Omega = 2\pi \times 1.6$ MHz, $d = 20\,\mu$m, and $N = 10$. Based on these estimations, a few traps (e.g., up to $N \sim 10$) could be integrated to realize a scaled quantum computing network, at least theoretically.

In conclusion, we have proposed an approach to realize the universal CNOT gate between the separately trapped ions. This approach operates beyond the usual LD limit and includes three steps laser operations and two steps adiabatic manipulations. The laser beams are used to couple the external and internal states of trapped ions, and the adiabatic manipulations are utilized to realize the switchable coupling between the vibrations of the ions. A possible method to implement the scalable ion-traps quantum computing networks is also presented.

Acknowledgements: This work is partly supported by the NSFC grant No. 10874142, 90921010, the grant from the Major State Basic Research Development Program of China (973 Program) (No. 2010CB923104), and the open project of state key laboratory of functional materials for informatics.

[1] P. W. Shor, in *Proceedings of the 35th Annual Symposium on Foundations of Computer Science*, edited by S. Goldwasser (IEEE Computer Society Press, Los Alamitos, CA, 1994), p. 124; A. Ekert and R. Jozsa, Rev. Mod. Phys. **68**, 733 (1996).
[2] J. I. Cirac and P. Zoller, Phys. Rev. Lett. **74**, 4091 (1995).
[3] J. M. Raimond, M. Brune, and S. Haroche, Rev. Mod. Phys. **73**, 565 (2001); J. I. Cirac, P. Zoller, H. J. Kimble, and H. Mabuchi, Phys. Rev. Lett. **78**, 3221 (1997); M. Keller *et al.*, Nature **431**, 1075 (2004).
[4] N. A. Gershenfeld and I. L. Chuang, Science **275**, 350 (1997).
[5] Y. Makhlin, G. Schön, and A. Shnirman, Rev. Mod. Phys. **73**, 357 (2001).
[6] D. Loss and D. P. DiVincenzo, Phys. Rev. A **57**, 120 (1998).
[7] D. Leibfried, R. Blatt, C. Monroe, and D. Wineland, Rev. Mod. Phys. **75** 281 (2003); H. Häffner *et al.*, Nature **438**, 643 (2005).
[8] G. J. Milburn, S. Schneider, and D. F. V. James, Fortschr. Phys. **48** 801 (2000).
[9] K. Mølmer and A. Sørensen, Phys. Rev. Lett. **82** 1835 (1999).
[10] J. I. Cirac and P. Zoller, Nature **404**, 579 (2000).
[11] D. Kielpinski, C. Monroe, and D. J. Wineland, Nature **417**, 709 (2002).
[12] K. R. Brown *et al.*, Nature **471**, 196 (2011).
[13] M. Harlander *et al.*, Nature **471**, 200 (2011).
[14] T. Sleator and H. Weinfurter, Phys. Rev. Lett. **74**, 4087 (1995); D. P. DiVincenzo, Phys. Rev. A **51**, 1015 (1995).
[15] L. F. Wei, S. Y. Liu, and X. L. Lei, Phys. Rev. A, **65** 062316 (2002); L. F. Wei, Y. X. Liu, and F. Nori, Phys. Rev. A, **70** 063801 (2004); L. F. Wei, M. Zhang, H. Y. Jia, and Y. Zhao, Phys. Rev. A, **78** 014306 (2008).
[16] M. Zhang, H. Y. Jia, and L. F. Wei, Optics Communications, **282** 1948 (2009); C. Monroe *et al.*, Phys. Rev. A **55**, R2489 (1997).
[17] D. M. Meekhof *et al.*, Phys. Rev. Lett. **76**, 1796 (1996).
[18] C. Monroe *et al.*, Phys. Rev. Lett. **75**, 4714 (1995).